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POSITIVE SOLUTIONS WITH PRESCRIBED PATTERNS IN SOME SIMPLE SEMILINEAR EQUATIONS

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1. Introduction

Recently, for various elliptic problems of the type

$$-\epsilon\Delta u = f(u), \quad x \in \Omega;\ Bu = 0,\ x \in \partial\Omega$$

where $Bu = u$ (Dirihclet boundary conditions) or $Bu = \partial u/\partial\nu$ (Neumann boundary conditions), it has been proved that positive solutions with a single sharp peak or multiple sharp peaks exist when $\epsilon > 0$ is sufficiently small. See, for example, [5, 12, 14] and the references therein. Since the space variable x does not appear in the nonlinearity, the topology and geometry of the domain Ω plays a central role in these problems.

For problems of the type

$$-\Delta u = f_\epsilon(x, u),\ u|_{\partial\Omega} = 0,$$

where

$$f_\epsilon(x, u) = Q(x)u^{\frac{N+2}{N-2}} + \epsilon u \quad (\text{ see, e.g., [3]}),$$

or

$$f_\epsilon(x, u) = Q(x)u^p - \epsilon^{-1}u \quad (\text{ see, e.g., [4]}),$$

or

$$f_\epsilon(x, u) = \epsilon^{-1}[u^p + Q(x)u] \quad (\text{ see, e.g., [10]}),$$

with $1 < p < (N+2)/(N-2)$, $\Omega \subseteq R^N$ $(N \geq 3)$, single and multiple peaked positive solutions have also been established for small positive ϵ. Here the peaks are determined largely by the function $Q(x)$.

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