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ON SOME QUASILINEAR PDE'S WITH SINGULARITIES ON THE BOUNDARY

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This paper deals with the problem of existence of nonnegative solutions to quasilinear elliptic equations and inequalities with Dirichlet boundary conditions, as well as to their evolution counterparts. A typical example of the former could be the problem

$$\begin{cases} -\Delta_p u - \lambda a(x) u^{p-1} = b(x) u^\beta \delta^{-\alpha}(x) & (x \in \Omega), \\ u(x) \geq 0 & (x \in \Omega), \\ u(x) = 0 & (x \in \partial\Omega). \end{cases} \tag{0.1}$$

Here Ω is a smooth bounded domain in $\mathbb{R}^n$, $1 < p < n$, $\lambda \in \mathbb{R}$ is a spectral parameter, $a \in L^\infty(\Omega)$ and $b \in C(\overline{\Omega})$ are nonnegative and not identically zero (unless specified differently, as in Theorems 1.2–1.4), and the operator Δ_p is defined, as usual, by the formula $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$. Besides being a convenient model for degenerate elliptic equations in general, this operator has a number of interesting applications in dynamics of non-Newtonian fluid flows, flows through porous media, and glaciology (see [5]). However, some of our results apply to more general elliptic operators, such as those of mean curvature type. By $\delta(x) = \mathrm{dist}(x, \partial\Omega)$ we denote the distance from the point x to the boundary $\partial\Omega$. The range of parameters α and β will be specified below.

For $\alpha \leq 0$, this problem belongs to a class, which was thoroughly studied in [6]. The case $\alpha > 0$ presents additional difficulties, since this hypothesis means that the nonlinearity coefficient is singular on the boundary. Nevertheless, in the semilinear case $p = 2$ and $a(x) \equiv 0$ (that is, for the usual Laplacian), this problem was also considered by a number of authors who employed ODE and variational methods (see [10], [11], [19]). Under their

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