Differential and Integral Equations Volume 16, Number 1, January 2003, Pages 13–50

EXISTENCE FOR A NONLINEAR WAVE EQUATION WITH DAMPING AND SOURCE TERMS

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1. Introduction

We study the Cauchy problem

$$u_{tt} - \Delta u + Q(t, x, u_t) = f(x, u), \quad u(0) = u_0,\ u_t(0) = u_1, \tag{1.1}$$

in $(0, T) \times \mathbb{R}^n$, where $0 < T \leq \infty$, $n \geq 1$. The data u_0 and u_1 are in the energy space and are compactly supported in $\mathbb{R}^n$. The terms Q and f are nonlinear damping and source terms respectively.

Problems like (1.1) have been studied in many papers. For the case $Q \equiv 0$ see [3], [7], [11], [12], [20], [30]; for the case $f \equiv 0$ see [8], [10]; when both terms Q and f interact, see [6], [9], [13], [14], [15], [18], [19], [25], [26], [27], [31], [32], [33].

For the sake of simplicity we illustrate our results by considering the model power–like equation with damping term

$$Q(x, v) = \sigma(x)|v|^{m-2}v, \tag{1.2}$$

where $m > 1$, σ is a measurable function such that $c_1 \leq \sigma(x) \leq c_2$ on $\mathbb{R}^n$, c_1, c_2 are positive constants, and the source term is in the form

$$f(x, u) = \mu_1(x)|u|^{p-2}u + \mu_2(x)|u|^{q-2}u, \tag{1.3}$$

with $p > 2$, $1 < q < p$, and $\mu_i \in L^\infty_{\rm loc}(\mathbb{R}^n)$, $i = 1, 2$.

Accepted for publication: June 2002.
AMS Subject Classifications: 34D05, 70D10, 94C99.