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LINEAR ELLIPTIC SYSTEMS INVOLVING FINITE RADON MEASURES

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1. Statement of the main result

The study of elliptic boundary value problems with L^1 or Radon measure data has been initiated in the last few decades by the pioneering works of Stampacchia [12], Brezis-Strauss [7], Brezis [5], [6].

Let Ω be a smooth bounded domain in $\mathbf{R}^N$. Consider the problem

$$\begin{cases} -\text{div } (a_i(x)\nabla u_i) + \displaystyle\sum_{j=1}^{d} b_{ij}(x)u_j = f_i, \quad \text{in } \Omega, \text{ for } i = 1, \cdots, d \\ u_i = 0, \quad \text{on } \Gamma_{\mathcal{D}}, \text{ for } i = 1, \cdots, d \\ \dfrac{\partial u_i}{\partial \nu} = g_i, \quad \text{on } \Gamma_{\mathcal{N}}, \quad \text{for } i = 1, \cdots, d\,. \end{cases} \tag{1.1}$$

Here, ν denotes the unit normal outward vector, $d \geq 1$ is an integer, and $a_i, b_{ij} \in L^\infty(\Omega)$, for $1 \leq i, j \leq d$. We point out that we make no symmetry assumption on the coefficients b_{ij}. We assume that $\{\Gamma_{\mathcal{D}}, \Gamma_{\mathcal{N}}\}$ realize an open partition of the boundary $\partial\Omega$, i.e., $\Gamma_{\mathcal{D}} \cap \Gamma_{\mathcal{N}} = \emptyset$ and $\overline{\Gamma_{\mathcal{D}}} \cup \overline{\Gamma_{\mathcal{N}}} = \partial\Omega$. Moreover, we suppose that $\Gamma_{\mathcal{D}}$ has nonzero $(N-1)$-Lebesgue measure, namely, $meas_{N-1}(\Gamma_{\mathcal{D}}) > 0$. We also assume that the elliptic operator is not degenerate, i.e., there exists $\alpha > 0$ such that

$$a_i(x) \geq \alpha \qquad \text{for a.e. } x \in \Omega \text{ and any } i = 1, \cdots, d. \tag{1.2}$$

Set $E^{1,p}(\Omega) := \{u \in W^{1,p}(\Omega); u = 0 \text{ on } \Gamma_{\mathcal{D}}\}$ and $E := \bigcap_{1 \leq p < \frac{N}{N-1}} (E^{1,p}(\Omega))^d$. We denote throughout by $\|\cdot\|_p$ (resp. $\|\cdot\|_{p,d}$) the norm in the space $L^p(\Omega)$

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