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GLOBAL ASYMPTOTIC STABILITY FOR FINITE-CROSS-SECTION PLANAR SHOCK PROFILES OF VISCOUS SCALAR CONSERVATION LAWS

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1. Introduction

We consider a possibly multidimensional viscous scalar conservation law

$$u_t + \sum_{j=1}^{d} f_j(u)_{x_j} = \Delta u \tag{1.1}$$

for $(t, x) = (t, x_1, y) \in (0, \infty) \times \mathbb{R} \times \Omega_{d-1}$, where Ω_{d-1} is a smooth, bounded domain in $\mathbb{R}^{d-1}$, whose volume will be denoted $vol\Omega_{d-1}$. A planar shock profile for (1.1) is a solution ψ of that equation that depends only on the combination $x_1 - st$ for some constant s, and tends to distinct finite values $u_\pm$ as $x_1 \to \pm\infty$. That is,

$$-s\psi' + f_1(\psi)' = \psi'', \quad \lim_{x_1 \to \pm\infty} \psi(x_1) = u_\pm. \tag{1.2}$$

Note that if $\psi(x_1)$ satisfies (1.2) then so does the translate $\psi(x_1 - \delta)$ for any real δ.

As is well-known (e.g. [9]), integrating (1.2) shows that s, $u_\pm$, and f_1 must satisfy the Rankine-Hugoniot condition

$$s = \frac{f_1(u_+) - f_1(u_-)}{u_+ - u_-} \tag{1.3}$$

and the entropy condition

$$sgn(u_+ - u_-)\big\{\big[f_1(u) - su\big] - \big[f_1(u_\pm) - su_\pm\big]\big\} > 0 \tag{1.4}$$

for all u between u_+ and u_-. The Rankine-Hugoniot condition (1.3) implies that the same condition is obtained for both choices of the sign $\pm$ in (1.4). It is also well-known that taking the limit of (1.4) as $u \to u_\pm$ shows that

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