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INTEGRAL EQUATION METHODS FOR DIV-CURL PROBLEMS FOR PLANAR VECTOR FIELDS IN NONSMOOTH DOMAINS

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1. Introduction and statement of main results

A classical problem arising in fluid dynamics and electrodynamics is the determination of a vector field with prescribed divergence and curl. Such a vector field can be considered in the whole plane/space or on domains with boundary when either its tangential component (Dirichlet condition) or its normal component (Neumann condition) is prescribed on the boundary. These problems are usually referred to as div-curl problems and have been studied in the literature both from an analytical point of view (see, e.g., [2], [12], [21]), as well as from a numerical analysis perspective (cf., e.g., [6], [18], [19], [20]), [17]. A nice exposition, which also deals with the case of mixed boundary conditions, can be found in [1].

In the broader context of partial differential equations, the importance of the div-curl system stems from the fact that this is the most basic linear elliptic system and, as such, it has stimulated a substantial body of work, particularly in relation to regularity aspects. Quite recently, some new, deep endpoint regularity results for div-curl problems have been announced by J. Bourgain and H. Brezis in [5].

In this paper, we treat planar div-curl problems, with prescribed Dirichlet or Neumann boundary conditions, on an arbitrary bounded domain with Lipschitz boundary. We are concerned with the regularity of the solution measured on Sobolev scales $H^{s,p}$, globally defined in the domain in question. In this analytic-geometric context, we derive estimates valid for optimal ranges of the exponents involved (p for integrability, and s for smoothness)

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