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POSITIVE SOLUTIONS FOR A CLASS OF SINGULAR SEMIPOSITONE $p-$LAPLACIAN PROBLEMS

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1. Introduction

In this paper, we consider the existence of positive solutions to the boundary-value problems

$$\begin{cases} (q(t)\phi(u'))' + \lambda r(t) f(u) = 0, \quad t \in (a,b) \\ u(a) = u(b) = 0, \end{cases} \tag{1.1}$$

where $\phi(x) = |x|^{p-2}x, p > 1,\ q, r : [a,b] \to [0,\infty),\ f : (0,\infty) \to \mathbb{R}$ may have negative values and may become infinite at 0, and λ is a positive parameter.

When $q(t) = t^{N-1}$, (1.1) occurs in the study of radial solutions for the $p-$Laplacian boundary-value problem

$$\begin{cases} \Delta_p u + \lambda r(|x|) f(u) \ \text{ in } \ \Omega \\ u = 0 \ \text{ on } \partial\Omega, \end{cases} \tag{1.2}$$

where $\Delta_p u \equiv div(|\nabla u|^{p-2}\nabla u)$ and Ω is the annulus $0 < a < |x| < b,\ x \in \mathbb{R}^N$. We refer to the survey article [3] for the literature on this problem when f is nonsingular. The case when $p = 2$ and f is possibly singular at 0 with negative values was considered in [1,5]. In [5], Liu obtained the existence of at least two positive solutions to the problem

$$\begin{cases} u'' + \lambda f(t,u) = 0, \quad t \in (0,1) \\ u(0) = u(1) = 0, \end{cases}$$

for $\lambda > 0$ small, where f satisfies, among other assumptions, $f(t,u) + M \geq 0$ for all $(t,u) \in (0,1) \times (0,\infty)$, for some $M > 0$, and $\lim s_{u \to 0^+} f(t,u) = \infty$,

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