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LOCAL HESSIAN ESTIMATES FOR SOME CONFORMALLY INVARIANT FULLY NONLINEAR EQUATIONS WITH BOUNDARY CONDITIONS

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1. Introduction

Consider a smooth compact Riemannian manifold $(\mathcal{M}^n, g)$ of dimension $n \geq 3$ with smooth boundary $\partial\mathcal{M}$. Let h_g be the mean curvature of $\partial\mathcal{M}$ with respect to g, and denote by $Riem_g$, Ric_g and R_g the Riemannian curvature tensor, the Ricci tensor and the scalar curvature of g, respectively. Then one has the standard decomposition

$$Riem_g = W_g + A_g \odot g,$$

where W_g is the Weyl curvature tensor of g, $\odot$ denotes the Kulkarni-Nomizu product, and A_g is the Schouten tensor of g defined as

$$A_g := \frac{1}{n-2}\Big(Ric_g - \frac{1}{2(n-1)} R_g g\Big).$$

Since the Weyl tensor is conformally invariant, the transformation of the Riemannian curvature tensor under conformal change of metrics is determined by the transformation of the Schouten tensor. It is therefore of interest to study the curvature functions of the Schouten tensor under conformal deformation.

Let f be a positive symmetric function defined on an open convex symmetric cone Γ with vertex at the origin. For a given number $c \in \mathbb{R}$, one interesting problem is to find a metric $\tilde{g}$ conformal to g such that

$$\begin{cases} f(\lambda(A_{\tilde{g}})) = 1, \quad \lambda(A_{\tilde{g}}) \in \Gamma \quad \text{on } \mathcal{M}, \\ h_{\tilde{g}} = c \quad \text{on } \partial\mathcal{M}, \end{cases} \tag{1.1}$$

where $\lambda(A_{\tilde{g}})$ denote the eigenvalues of $A_{\tilde{g}}$ with respect to $\tilde{g}$.

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