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VISCOSITY SUPERSOLUTIONS OF THE EVOLUTIONARY p-LAPLACE EQUATION

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1. Introduction

Often new proofs of old results give additional insight, besides the simplification offered. We hope that the present study of the diffusion equation

$$\frac{\partial v}{\partial t} = \nabla \cdot (|\nabla v|^{p-2} \nabla v) \tag{1.1}$$

has this character. Even obvious results for this equation may require advanced estimates in the proofs. We refer to the books [3] and [13] about this equation, which is called the "evolutionary p-Laplacian equation," the "p-parabolic equation" or even the "non-Newtonian equation of filtration."

Our objective is to study the regularity of the *viscosity supersolutions* and their spatial gradients. We give a new proof of the existence of ∇v in Sobolev's sense and of the validity of the equation

$$\iint_\Omega \Big(-v \frac{\partial \varphi}{\partial t} + \langle |\nabla v|^{p-2} \nabla v,\ \nabla \varphi \rangle \Big) dx\ dt \geq 0 \tag{1.2}$$

for all test functions $\varphi \geq 0$. Here Ω is the underlying domain in $\mathbb{R}^{n+1}$ and v is a bounded viscosity supersolution in Ω. The first step of our proof is to establish (1.2) for the so-called infimal convolution v_ϵ, constructed from v through a simple formula. The function v_ϵ has the advantage of being differentiable with respect to all its variables $x_1, x_2, \ldots, x_n$, and t, while the

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