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CONSTRUCTION OF SINGULAR SOLUTIONS FOR ELLIPTIC PROBLEM OF FOURTH ORDER DERIVATIVE WITH A SUBCRITICAL NON-LINEARITY

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1. Introduction and statement of the results

Many authors have studied the existence of weak solutions for the semi-linear elliptic equation

$$\Delta u + u^p = 0, \tag{1.1}$$

which are positive in a domain $\Omega \subset \mathbb{R}^n$, and prescribed isolated singularities or singular along arbitrary smooth submanifolds for special values of the exponent p.

The asymptotic behavior near an isolated singularity has been studied by Aviles in [2] when $p = \frac{n}{n-2}$, by Gidas and Spruck in [8], when $p \in (\frac{n}{n-2}, \frac{n+2}{n-2})$, and finally by Caffarelli, Gidas and Spruck in their paper [4], for the case $p = \frac{n+2}{n-2}$, which is the so-called critical exponent. They give some results about the asymptotic behavior of the singular solutions of (1.1).

Concerning the existence of weak solutions to the equation (1.1), which are positive in $\Omega \subset \mathbb{R}^n$, vanish at the boundary, it is known that if the exponent p is less than $\frac{n}{n-2}$, then any weak solution must be smooth on all of Ω. The existence of solutions of this equation with prescribed isolated singularities when p lies in the interval $\frac{n}{n-2} \leq p < p_0$, where p_0 is some value close to $\frac{n}{n-2}$ and in particular, less than $\frac{n+2}{n-2}$, has already been solved by Pacard in [15] and [16]. When $p = \frac{n+2}{n-2}$, the problem becomes conformally invariant. There is then a loss of compactness, and the problem consequently

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