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ON A CLASS OF 1-MONOTONE SOLUTIONS FOR A FORCED PENDULUM MODEL

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Dedicated to Jean Mawhin who always liked the pendulum

1. Introduction

As has been noted by Jean Mawhin [4], the pendulum has served as a paradigm for nonlinear analysis and dynamical systems. In a recent paper [1], elementary minimization arguments were used to find a variety of solutions of equations of forced pendulum type. The existence of these solutions is not new, but the approach is very simple and requires almost no facts from the theory of dynamical systems. The purpose of this note is to carry the analysis of [1] a step further to obtain another natural class of heteroclinic solutions. See the survey papers [3] and [4] by Mawhin for references and much more on the pendulum.

As in [1], the forced pendulum model equation

$$-u'' + V_u(x, u) = 0 \tag{1.1}$$

will be considered. Here V satisfies

- (V1) $V \in C^2(\mathbb{T}^2, \mathbb{R})$, $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$;

i.e., V is 1-periodic in its arguments. We further assume

- (V2) $V(-t, z) = V(t, z),\ t, z \in \mathbb{R}$;

i.e., V is time reversible. As in [1], (V2) is not necessary, but it considerably simplifies the construction of solutions of (1.1) that undergo transitions.

Before describing the new class of heteroclinics that will be obtained for (1.1), we recall some results from [1] for (1.1). Let $E = W^{1,2}(\mathbb{T}^1)$ be the Hilbert space of 1-periodic functions for which $||u|| < \infty$, where

$$||u||^2 = \int_0^1 (|u'|^2 + u^2)dt.$$

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