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THE PENDULUM EQUATION: FROM PERIODIC TO ALMOST PERIODIC FORCINGS

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(Dedicated to Patrick Habets and Jean Mawhin)

1. Introduction

Consider the differential equation

$$\ddot{x} + a \sin x = p(t), \tag{1.1}$$

where $a > 0$ is a parameter and $p : \mathbb{R} \to \mathbb{R}$ is an almost periodic function. Almost periodicity will be understood in the classical sense defined by Bohr [7]. The existence of almost periodic solutions has already been discussed in several papers by Blot [5], Mawhin [19, 20] and Belley and Saadi Drissi [2]. Also, the papers by Fink [10] and Fournier, Szulkin and Willem [12] contain results applicable to (1.1). In all these works there is some restriction on the size of the forcing. This size is measured with respect to different norms of p, always with the intention of locating the solution on an interval where the sine function is decreasing, say $(\frac{\pi}{2}, \frac{3\pi}{2})$. The possible novelty of the present paper is that it searches for results valid for forcings of arbitrary size. There are many other papers on the forced pendulum equation but they deal with the periodic case. See [20] for a recent survey. A nice feature of the periodically forced pendulum is that most of the methods of nonlinear analysis can be applied and lead to interesting conclusions. In this sense, the equation (1.1) becomes a good illustration for nonlinear mathematics. The almost periodic case is attractive just for opposite reasons. It seems that the standard techniques[1] are not applicable and that new phenomena appear. This is my main motivation for the present study but future applications in other fields cannot be discarded. Recently, I read an interesting paper by Futakata and Iwasaki on animal locomotion. In [13] a neuronal circuit, the so-called central pattern generator, is coupled with a forced pendulum

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[1] Variational methods, continuation and degree theory, upper and lower solutions