

SMALL AMPLITUDE OSCILLATORY SHEAR PERMEATION FLOW OF CHOLESTERIC LIQUID CRYSTAL POLYMERS*

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Abstract. We investigate the small amplitude oscillatory shear permeation flow (SAOSPF) of cholesteric liquid crystal polymers (CLCPs) using a mesoscopic model obtained from the Doi kinetic theory for flows of CLCPs. We model the system by Stokes hydrodynamic equations coupled with orientational dynamics and study the frequency-locked solutions of the system by employing a coarse-grained approach. The leading order solutions exhibit boundary layers of thickness of the order of the cholesteric pitch in low frequency regimes but in the order of $\frac{1}{\sqrt{\omega}}$ in high frequency regimes, where ω is the plate driven frequency. The response of CLCPs has been calculated. Viscoelastic behavior is only observed at intermediate frequencies and the material is essentially viscous at low and high frequency regimes. At both low and high frequencies, the real component of complex viscosity controls its magnitude which is of the order of the reciprocal of the cholesteric pitch at low frequencies while the order of 1 at high frequencies.

Key words. Asymptotic expansion, cholesteric liquid crystals, permeation flows, hydrodynamics.

AMS subject classifications. 76A15, 76M45.

1. Introduction

Cholesteric liquid crystals (CLCs) are mesophases between liquids and solids, in which the average molecular orientation exhibits a chiral (twisted) orientational pattern in a direction normal to the local averaged molecular orientation, which is known as the chiral helix [5, 12]. Cholesterol esters, DNAs, colloidal suspensions of bacteriophages, and many biological materials are examples that can exhibit chiral phases under proper conditions. The chiral liquid crystal phase is delicate in that it can be sustained only under balanced conditions; otherwise, defects and disclinations can form easily. As a result, there are few quantitative experiments on the dynamics and rheology. The few available studies on flows of cholesterics are mostly based on Leslie-Ericksen (LE) continuum theory, which is remarkably successful in describing the dynamical behavior of low molecular-weight liquid crystals but not quite so for flows of liquid crystal polymers. Some studies based on perturbation techniques with the base helical axis in the flow direction [14, 15, 20, 16, 12, 5, 30], in the velocity gradient direction [19, 17], and in the vorticity direction in simple shear [13, 24, 25, 26] using the LE theory are available. Brief reviews on cholesteric rheology can be found in the books [12, 18].

Liquid crystals exhibit a viscoelastic response to an external stress. Coupling between the director and the velocity fields — known as backflow — leads to strongly non-Newtonian flow behavior. A particular striking example in flows of cholesterics is permeation where a cholesteric liquid crystal is subjected to an imposed flow in the direction of the helix but the helical structure remains nearly intact. The main feature of the permeation is its ultrahigh viscosity (by a factor of $\sim 10^5$ of pure nematic liquid crystals) in small shear. But, in strong shear, the helical structure is completely destroyed so that cholesterics behave essentially like pure nematic liquid crystals. An

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