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TAILORED FINITE POINT METHOD FOR STEADY-STATE REACTION-DIFFUSION EQUATIONS*

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Abstract. In this paper, we propose to use the tailored-finite-point method (TFPM) for a type of steady-state reaction-diffusion problems in two dimensions. Three tailored finite point schemes are constructed for the given problem. Our finite point method has been tailored to some particular properties of the problem. Therefore, our TFPM satisfies the discrete maximum principle automatically. We also study the error estimates of our TFPM. We prove that our TFPM can achieve good accuracy even when the mesh size $h \gg \varepsilon$. Our numerical examples show the efficiency and reliability of our method.

Key words. Tailored finite point method, singular perturbation problem, boundary layer, discrete maximum principle.

AMS subject classifications. 65N30, 35J65.

1. Introduction

We consider the steady-state reaction-diffusion equation in the unit square $\Omega = (0,1)\times(0,1)$:

$$\mathbf{L}u \equiv -\varepsilon^2 \triangle u + b(x,y)u = f(x,y), \quad \text{in} \quad \Omega, \tag{1.1}$$

$$u = 0, \quad \text{on} \quad \Gamma = \partial\Omega, \tag{1.2}$$

where $b(x,y)$ and $f(x,y)$ are two given functions on $\bar{\Omega}$ and

$$b(x,y) \geq b_{min} > 0, \text{ on } \bar{\Omega}.$$

Furthermore we suppose that the given functions $b(x,y), f(x,y) \in \mathbf{C}^{4,\beta}(\bar{\Omega})$ for a real number $\beta \in (0,1)$, and the function $f(x,y)$ satisfies the corner compatibility conditions:

$$f(0,0) = f(1,0) = f(0,1) = f(1,1) = 0. \tag{1.3}$$

Then we know that the solution of problem (1.1)–(1.2), $u(x,y) \in \mathbf{C}^{6,\beta}(\Omega) \cap \mathbf{C}^{3,\beta}(\bar{\Omega})$ [5].

The problem (1.1)–(1.2) is a singular perturbation problem when $\varepsilon \ll 1$; the solution of problem (1.1)–(1.2) is allowed boundary layers as well as corner layers [10]. These layers are characterized by rapid transitions in the solution, and are thus difficult to capture in a numerical approximation without using a large number of unknowns. Also, such layers tend to cause spurious oscillations in a numerical solution to the problem.

Methods for the numerical solution of problems such as (1.1)–(1.2) in bounded or unbounded domains that attempt to deal with these difficulties have been developed by many mathematicians, see e.g., [2, 3, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25].

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