Commun. Math. Sci.
Vol. 8, No. 4, pp. 851–862

A FINITE TIME RESULT FOR VANISHING VISCOSITY IN THE PLANE WITH NONDECAYING VORTICITY*

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Abstract. Assuming that initial velocity has finite energy and initial vorticity is bounded in the plane, we show that the unique solutions of the Navier-Stokes equations converge to the unique solution of the Euler equations in the L^∞-norm uniformly over finite time as viscosity approaches zero. We also establish a rate of convergence.

Key words. Fluid mechanics, inviscid limit.

AMS subject classifications. 76D05, 76C99.

1. Introduction

We consider the Navier-Stokes equations modeling incompressible viscous fluid flow, given by

$$(NS)\begin{cases}\partial_t v_\nu + v_\nu\cdot\nabla v_\nu - \nu\Delta v_\nu = -\nabla p_\nu\\ \text{div } v_\nu = 0\\ v_\nu|_{t=0} = v_\nu^0,\end{cases}$$

and the Euler equations modeling incompressible non-viscous fluid flow, given by

$$(E)\begin{cases}\partial_t v + v\cdot\nabla v = -\nabla p\\ \text{div } v = 0\\ v|_{t=0} = v^0.\end{cases}$$

In this paper, we study the vanishing viscosity limit. The question of vanishing viscosity addresses whether or not a solution v_ν of (NS) converges in some norm to a solution v of (E) with the same initial data as viscosity tends to 0.

While there exist many vanishing viscosity results in the plane for weak solutions to the fluid equations, the most relevant result for our purposes is that of Chemin in [1] concerning Yudovich solutions to (E). Specifically, in [8], Yudovich establishes the uniqueness of a solution (v,p) to (E) in the space $C(\mathbb{R}^+;L^2(\mathbb{R}^2))\times L^\infty_{loc}(\mathbb{R}^+;L^2(\mathbb{R}^2))$ when v^0 belongs to $L^2(\mathbb{R}^2)$ and ω^0 belongs to $L^p(\mathbb{R}^2)\cap L^\infty(\mathbb{R}^2)$ for some $p<\infty$. For this uniqueness class, Chemin proves that the vanishing viscosity limit holds in the L^p-norm uniformly over finite time, and he establishes a rate of convergence.

In this paper, we consider the case where initial velocity belongs to $L^2(\mathbb{R}^2)$, while initial vorticity is bounded and does not necessarily belong to $L^p(\mathbb{R}^2)$ for any $p<\infty$. The existence and uniqueness of solutions to (E) with nondecaying vorticity *and* nondecaying velocity was proved by Serfati in [7]. Specifically, the author proves the following theorem.

Theorem 1.1. *Let v^0 and ω^0 belong to $L^\infty(\mathbb{R}^2)$, and let $c\in\mathbb{R}$. For every $T>0$ there exists a unique solution (v,p) to (E) in the space* $L^\infty([0,T];L^\infty(\mathbb{R}^2))\times$

*Received: April 8, 2009; accepted (in revised version); November 4, 2009. Communicated by Francois Golse.
This material is based upon work supported by the Center for Nonlinear Analysis (CNA) under the National Science Foundation Grant No. DMS 0635983.
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