COMMUN. MATH. SCI.
Vol. 8, No. 4, pp. 835–850

SOME DECAY ESTIMATES OF SOLUTIONS FOR THE 3-D COMPRESSIBLE ISENTROPIC MAGNETOHYDRODYNAMICS*

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Abstract. This paper is concerned with the time-asymptotic behavior of solutions to the three-dimensional magnetohydrodynamics (MHD) for viscous compressible isentropic fluids. By exploiting some L^p-L^q estimates of solutions for the heat equation and the linearized Navier-Stokes system, the optimal decay estimates of the solution in L^q with $2 \le q \le 6$ and its first order derivative in L^2 are obtained when the initial perturbation around a constant state is sufficiently small in H^3 and is bounded in L^p with any given $1 \le p < 6/5$. As a byproduct, the global existence theorem is also proved.

Key words. compressible isentropic MHD, decay estimates, L^p-L^q estimates, global existence.

AMS subject classifications. 76W05, 76N10, 35B40, 35B45.

1. Introduction

Magnetohydrodynamics (MHD) concerns the motion of a conducting fluid (plasma) in an electromagnetic field with a very wide range of applications. Due to the interaction between the dynamic motion of the fluid and the evolutions of the magnetic field, the hydrodynamic and electrodynamic effects are strongly coupled. The governing equations for three-dimensional compressible magnetohydrodynamic flows, derived from fluid mechanics with appropriate modifications to account for electrical forces, have the following form (see, e.g., [3, 15, 30, 31, 42]):

$$\begin{cases} \rho_t + \operatorname{div}(\rho \boldsymbol{u}) = 0, \\ (\rho \boldsymbol{u})_t + \operatorname{div}(\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p(\rho) = (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \lambda \Delta \boldsymbol{u} + (\lambda + \lambda') \nabla \operatorname{div} \boldsymbol{u}, \\ \boldsymbol{B}_t - \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) = -\nabla \times (\nu \nabla \times \boldsymbol{B}), \qquad \operatorname{div} \boldsymbol{B} = 0, \end{cases} \tag{1.1}$$

where the unknown functions ρ, $\boldsymbol{u} \in \mathbb{R}^3$, $\boldsymbol{B} \in \mathbb{R}^3$, and $p = p(\rho)$ are the density, the velocity, the magnetic field, and the pressure of flows, respectively, λ and λ' are the viscosity coefficients of flows satisfying $\lambda > 0$ and $2\lambda/3 + \lambda' \ge 0$, and $\nu > 0$ is the resistivity constant acting as the magnetic diffusion coefficient of magnetic field.

There have been a lot of studies on MHD by many physicists and mathematicians due to its physical importance, complexity, rich phenomena, and mathematical challenges; see, for example, [3]–[5], [11]–[13], [15], [18]–[20], [23]–[26], [30, 31], [41]–[43], and the references cited therein. In particular, when there is no electromagnetic field, that is, $\boldsymbol{B} \equiv 0$, the system (1.1) reduces to the compressible Navier-Stokes equations for isentropic fluids, which have been studied by many researchers; see, for example, [14, 33, 36, 38] among others. Compared with the Navier-Stokes equations, the presence of magnetic field and its interaction with the hydrodynamic motion in MHD flows

*Received: July 3, 2009; accepted: November 4, 2009. Communicated by Zhouping Xin.
The first author was partly supported by the National Natural Science Foundation of China (Grant No. 10801111, 10601008) and the second author was partly supported by the National Natural Science Foundation of China (Grant No. 10971171).
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