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FAST LINEARIZED BREGMAN ITERATION FOR COMPRESSIVE SENSING AND SPARSE DENOISING*

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Dedicated to Andy Majda on his sixtieth birthday

Abstract. We propose and analyze an extremely fast, efficient, and simple method for solving the problem:

$$\min\{\|u\|_1 : Au = f, u \in R^n\}.$$

This method was first described in [J. Darbon and S. Osher, preprint, 2007], with more details in [W. Yin, S. Osher, D. Goldfarb and J. Darbon, SIAM J. Imaging Sciences, 1(1), 143-168, 2008] and rigorous theory given in [J. Cai, S. Osher and Z. Shen, Math. Comp., to appear, 2008, see also UCLA CAM Report 08-06] and [J. Cai, S. Osher and Z. Shen, UCLA CAM Report, 08-52, 2008]. The motivation was compressive sensing, which now has a vast and exciting history, which seems to have started with Candes, et. al. [E. Candes, J. Romberg and T. Tao, 52(2), 489-509, 2006] and Donoho, [D.L. Donoho, IEEE Trans. Inform. Theory, 52, 1289-1306, 2006]. See [W. Yin, S. Osher, D. Goldfarb and J. Darbon, SIAM J. Imaging Sciences 1(1), 143-168, 2008] and [J. Cai, S. Osher and Z. Shen, Math. Comp., to appear, 2008, see also UCLA CAM Report, 08-06] and [J. Cai, S. Osher and Z. Shen, UCLA CAM Report, 08-52, 2008] for a large set of references. Our method introduces an improvement called "kicking" of the very efficient method of [J. Darbon and S. Osher, preprint, 2007] and [W. Yin, S. Osher, D. Goldfarb and J. Darbon, SIAM J. Imaging Sciences, 1(1), 143-168, 2008] and also applies it to the problem of denoising of undersampled signals. The use of Bregman iteration for denoising of images began in [S. Osher, M. Burger, D. Goldfarb, J. Xu and W. Yin, Multiscale Model. Simul, 4(2), 460-489, 2005] and led to improved results for total variation based methods. Here we apply it to denoise signals, especially essentially sparse signals, which might even be undersampled.

Key words. ℓ_1-minimization, basis pursuit, compressed sensing, sparse denoising, iterative regularization.

AMS subject classifications. 49M99, 90-08, 65K10.

1. Introduction

Let $A \in R^{m\times n}$, with $n > m$ and $f \in R^m$, be given. The aim of a basis pursuit problem is to find $u \in R^n$ by solving the constrained minimization problem

$$\min_{u\in R^n}\{J(u) | Au = f\}, \tag{1.1}$$

where $J(u)$ is a continuous convex function.

For basis pursuit, we take:

$$J(u) = |u|_1 = \sum_{j=1}^{n} |u_j|. \tag{1.2}$$

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