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FAST INTERFACE TRACKING VIA A MULTIRESOLUTION REPRESENTATION OF CURVES AND SURFACES*

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Abstract. We consider the propagation of an interface in a velocity field. The initial interface is described by a normal mesh [Guskov, et al, SIGGRAPH Proc., 259-268, 2000] which gives us a multiresolution decomposition of the interface and the related wavelet vectors. Instead of tracking marker points on the interface we track the wavelet vectors, which like the markers satisfy ordinary differential equations. We show that the finer the spatial scale, the slower the wavelet vectors evolve. By designing a numerical method which takes longer time steps for finer spatial scales we are able to track the interface with the same overall accuracy as when directly tracking the markers, but at a computational cost of $O(\log N/\Delta t)$ rather than $O(N/\Delta t)$ for N markers and timestep Δt. We prove this rigorously and give numerical examples supporting the theory. We also consider extensions to higher dimensions and co-dimensions.

Key words. Interface tracking, multiresolution analysis, normal meshes, fast algorithms.

AMS subject classifications. 65L20, 42C40, 65D10.

1. Introduction

Tracking the evolution of interfaces or fronts is important in many applications, for instance, wave propagation, multiphase flow, crystal growth, melting, epitaxial growth and flame propagation. The interface in these cases is a manifold of co-dimension one which moves according to some physical law that depends on the shape and location of the interface. We suppose for convenience that it can be parameterized, so that for a fixed time t the interface is described by the function $x(t,s):\mathbb{R}^+\times\mathbb{R}^q\to\mathbb{R}^d$, with the parameterization $s\in\Omega\subset\mathbb{R}^q$ and $q=d-1$. In this paper we consider the simplified case when the interface is moving in a time-varying velocity field that does not depend on the shape of the front, only its location. Then $x(t,s)$ satisfies the parameterized ordinary differential equation (ODE)

$$\frac{\partial x(t,s)}{\partial t}=F(t,x(t,s)),\qquad x(0,s)=\gamma(s),\qquad s\in\Omega,\tag{1.1}$$

where $F(t,x):\mathbb{R}^+\times\mathbb{R}^d\to\mathbb{R}^d$ is a given function representing the velocity field and $\gamma(s):\mathbb{R}^q\to\mathbb{R}^d$ is the initial interface. We will mostly discuss curves in two dimensions, $d=2$, $q=1$, but we will also discuss extensions to higher dimensions $d=3$, $q=2$ and co-dimensions $d=3$, $q=1$. Applications could include the tracking of physically motivated interfaces, like wavefronts in high frequency wave propagation problems, or "artificial" fronts of propagation paths parameterized by initial data, where a problem has the structure of (1.1) even though the front has no direct physical interpretation. This could be, for instance, iso-distance curves on a surface (front of geodesics), fiber tract bundles in brain imaging, or the method of characteristics for the solution graph of hyperbolic partial differential equations (PDEs). In many of these problems it is better numerically to consider a front rather than a set of individual paths, since the connectivity between paths is then maintained, which for example simplifies interpolation between them.

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