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ON ONE VERSION OF A SEMIDISCRETE GALERKIN METHOD FOR PDE PROBLEMS INVOLVING A GENERALIZED 2D HAMILTONIAN OPERATOR*

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Abstract. We consider a boundary value problem, an eigenvalue problem and an initial-boundary value problem involving a generalized 2D Hamiltonian operator (i.e., the second order self-adjoint elliptic operator) in a rectangular domain. We apply a semi-discrete Galerkin method exploiting space approximations of the form $c_1(x_1)\chi_1(x_1,x_2)+\cdots+c_N(x_1)\chi_N(x_1,x_2)$, where $\chi_1,\ldots,\chi_N$ are some known complex-valued basis functions and $c_1,\ldots,c_N$ are unknown coefficients. The corresponding approximate problems are stated and their properties such as existence and uniqueness of solutions and bounds for them, positive definiteness of the related sesquilinear forms, etc. are analyzed. For a specific physically reasonable choice of the basis functions, error bounds of arbitrarily high orders are proved for all the listed problems.

Key words. Elliptic equation, boundary value problem, eigenvalue problem, generalized time-dependent Schrödinger equation, semi-discrete method, Galerkin method, error bounds.

AMS subject classifications. 65N30, 65M60, 81V35.

1. Introduction

The description of large amplitude collective motion in atomic nuclei, such as large-scale collective oscillations and nuclear fission remains one of the most challenging problems in contemporary nuclear physics. One way of tackling this kind of problem is to employ the so-called Generator Coordinate Method (GCM) [11]. This approach consists in first defining a set of static nuclear wave-functions $\Phi_{\mathbf{q}}(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)$ labeled by the composite index $\mathbf{q}=(q_1,q_2,\ldots,q_M)$, representing the different kinds of shapes the system is expected to adopt during time-evolution. These wave-functions usually are determined by means of self-consistent mean-field Hartree-Fock-like techniques using M constraints of the form $q_l=\langle\Phi_{\mathbf{q}}|\hat{Q}_l|\Phi_{\mathbf{q}}\rangle$. In most cases the operators $\hat{Q}_l$ are taken as the usual multipole operators of order l. The time-dependent wave-function of the system is then assumed to be the superposition of all static states $\Phi_{\mathbf{q}}$ weighted by a probability amplitude $f(q;t)$:

$$\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A;t)=\int f(q;t)\,\Phi_{\mathbf{q}}(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\,d^Mq. \tag{1.1}$$

The justification for this form of nuclear wave-function is similar to the Bohr-Oppenheimer approximation of molecular physics. It is assumed that the "collective" degrees of freedom described by the set $\mathbf{q}$ evolve with characteristic times τ_{coll} that are much larger than the one associated with the internal motion of nucleons: $\tau_{coll}\gg\tau_{int}\simeq 10^{-23}$ s. The GCM is therefore expected to be a good approximation in the low-energy regime ($E\lesssim 15$ MeV).

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