COMMUN. MATH. SCI.
Vol. 6, No. 3, pp. 695–710

AN ANALYSIS OF THE DARWIN MODEL OF APPROXIMATION TO MAXWELL'S EQUATIONS IN 3-D UNBOUNDED DOMAINS*

CAIXIU LIAO† AND LUNG-AN YING‡

Abstract. In this paper, we derive the Darwin model in 3-D unbounded domains by the decomposition of the vector fields; then we show that the Darwin model approximates Maxwell's equations up to the second order for the magnetic flux density, and to the third order for the electric field with respect to $\eta = \frac{\bar{v}}{c}$, where $\bar{v}$ is the characteristic velocity and c is the speed of light.

Key words. Darwin model, Maxwell's equations, 3-D unbounded domains, approximation model

AMS subject classifications. 35Q60, 35J25, 35L50

1. Introduction

It is known that there are more and more problems involving the solution of Maxwell's equations:

$$\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mu J, \tag{1.1}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0, \tag{1.2}$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon}\rho, \tag{1.3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.4}$$

where $\mathbf{E}$=$\mathbf{E}$(x,t), $\mathbf{B}$=$\mathbf{B}$(x,t) denote the electric field and the magnetic flux density respectively, and $\rho = \rho(x,t)$, $\mathbf{J} = \mathbf{J}(x,t)$ are the charge and current densities, respectively, which satisfy the charge conservation equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0. \tag{1.5}$$

The positive constants ε, μ are the electric permittivity and the magnetic permeability of a vacuum, respectively. They are related by

$$\varepsilon \mu c^2 = 1.$$

In many cases, there are many challenges in dealing with Maxwell's equations directly and the numerical resolution may be very expensive in terms of the computational cost. However, for some problems, e.g., the simulation of charged particle beams when no high frequency phenomenon or no rapid current change occurs, it is possible to use some simplified model which approximates Maxwell's equations and can be solved more economically. The Darwin model is such a simplified model.

*Received: October 30, 2007; accepted (in revised version): January 30, 2008. Communicated by Pierre Degond.

†School of Mathematical Sciences, Xiamen University, Xiamen, 361005, China and Department of Mathematics, South China Normal University, Guangzhou, Guangdong, China (liaocaixiu800919@163.com).

‡School of Mathematical Sciences, Xiamen University, Xiamen, 361005, China (yingla@xmu.edu.cn).