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GLOBAL EXISTENCE OF WEAK SOLUTIONS TO THE REGULARIZED HOOKEAN DUMBBELL MODEL *

LINGYUN ZHANG†, HUI ZHANG‡, AND PINGWEN ZHANG§

Abstract. We consider a regularized Hookean dumbbell model in dilute polymeric solutions. Compared with the classical model, this model here is more natural, in which appear a macro diffusive term $\varepsilon\triangle_x\psi$ and Friedrichs mollifiers with a parameter α. Based on a compactness argument, the global existence of weak solutions to this model is established in the framework of the Rothe method. By a rigorous limiting process $\varepsilon\to 0_+$, we also obtain the global existence of weak solutions to the reduced model with $\varepsilon=0$.

Key words. global existence, Hookean dumbbell model, dilute polymer solutions, Navier-Stokes equation, Fokker-Planck equation, Rothe method

AMS subject classifications. 76D03, 82C31, 82D60

1. Introduction

In this paper, we investigate the global existence of weak solutions to a regularized Hookean dumbbell model for dilute polymeric fluids. In dilute polymer solutions, the polymer coils rarely overlap, so the interactions among polymer chains can be neglected. The polymer chains can be modelled by dumbbells, each with two beads connected by a single spring. The configuration of the spring then specifies the conformation of the polymer.

Denoting by $\mathbf{u}$ the velocity and by p the pressure, the governing equations for the incompressible polymeric fluids are

$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla_{\mathbf{x}})\mathbf{u}-\nu\Delta_{\mathbf{x}}\mathbf{u}+\nabla_{\mathbf{x}}p=\nabla_{\mathbf{x}}\cdot\tau \quad \text{in } \Omega\times(0,T], \tag{1.1}$$

$$\nabla_{\mathbf{x}}\cdot\mathbf{u}=0 \quad \text{in } \Omega\times(0,T]. \tag{1.2}$$

Here Ω is a bounded open set in $\mathbb{R}^d, d=2$ or 3, $\nu>0$ is the viscosity of the solvent, and τ is an extra stress tensor, which takes the form

$$\tau=k\omega(\mathbf{C}(\psi)-\rho(\psi)\mathbf{I}) \quad \text{in } \Omega\times(0,T], \tag{1.3}$$

$$\mathbf{C}(\psi)=\int_D(\nabla_q U\otimes\mathbf{q})\psi(\mathbf{x},\mathbf{q},t)d\mathbf{q} \quad \text{in } \Omega\times(0,T], \tag{1.4}$$

$$\rho(\psi)=\int_D\psi(\mathbf{x},\mathbf{q},t)d\mathbf{q} \quad \text{in } \Omega\times(0,T]. \tag{1.5}$$

Here $\kappa,\omega>0$ denote the Boltzmann constant and the absolute temperature, respectively. $\mathbf{I}$ is the unit $d\times d$ tensor, U is the spring potential. This stress tensor τ

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†LMAM and School of Mathematical Sciences, Peking University, Beijing 100871, P.R. China (lingyun@pku.edu.cn).

‡School of Mathematical Sciences, Beijing Normal University, Beijing 100875, P.R. China (hzhang@bnu.edu.cn). This work is supported in part by an Alexander von Humboldt Fellowship at Max-Planck Institute for Mathematical Science in Leipzig, NSFC for Project 10401008 and the key basic research project of the Ministry of Chinese Education 107016.

§LMAM and School of Mathematical Sciences, Peking University, Beijing 100871, P.R. China (pzhang@pku.edu.cn). This work is supported in part by the state key basic research project of China 2005CB321704 and NSFC for Distinguished Young Scholars 10225103.