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GLOBAL WELL-POSEDNESS OF THE THREE-DIMENSIONAL VISCOUS AND INVISCID SIMPLIFIED BARDINA TURBULENCE MODELS*

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Abstract. In this paper we present analytical studies of three-dimensional viscous and inviscid simplified Bardina turbulence models with periodic boundary conditions. The global existence and uniqueness of weak solutions to the viscous model has already been established by Layton and Lewandowski. However, we prove here the global well-posedness of this model for weaker initial conditions. We also establish an upper bound to the dimension of its global attractor and identify this dimension with the number of degrees of freedom for this model. We show that the number of degrees of freedom of the long-time dynamics of the solution is of the order of $(L/l_d)^{12/5}$, where L is the size of the periodic box and l_d is the dissipation length scale – believed and defined to be the smallest length scale actively participating in the dynamics of the flow. This upper bound estimate is smaller than those established for Navier-Stokes-α, Clark-α and Modified-Leray-α turbulence models which are of the order $(L/l_d)^3$. Finally, we establish the global existence and uniqueness of weak solutions to the inviscid model. This result has an important application in computational fluid dynamics when the inviscid simplified Bardina model is considered as a regularizing model of the three-dimensional Euler equations.

Key words. turbulence models, sub-grid scale models, large eddy simulations, global attractors, inviscid regularization of Euler equations.

MSC Classification. 35Q30, 37L30, 76BO3, 76D03, 76F20, 76F55, 76F65

1. Introduction

Let us denote by $v(x,t) = (v_1(x,t), v_2(x,t), v_3(x,t))$ the velocity field of an incompressible fluid and $p(x,t)$ its pressure. The three-dimensional (3D) Navier-Stokes equations (NSE)

$$\begin{aligned} \partial_t v - \nu\Delta v + \nabla\cdot(v\otimes v) &= -\nabla p + f, \\ \nabla\cdot v &= 0, \\ v(x,0) &= v^{in}(x), \end{aligned} \tag{1.1}$$

governs the dynamics of homogeneous incompressible fluid flows, where $f(x) = (f_1(x), f_2(x), f_3(x))$ is the body force assumed, for simplicity, to be time independent. The existing mathematical theory and techniques are not yet sufficient to prove the global well-posedness of the 3D NSE. Researchers who are investigating this question have incorporated the use of computers to analyze the dynamics of turbulent flows by studying the direct numerical simulation (DNS) of these equations. However, this is still a prohibitively expensive task to perform even with the most technologically advanced state-of-the-art computing resources. Tracking the pointwise flow values by numerical simulation for large Reynolds number is not only difficult but also, in some cases, disputable due to sensitivity of numerical solutions to perturbation errors in the data and the limitations of reliable numerical resolution. In many practical

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