COMM. MATH. SCI.
Vol. 3, No. 4, pp. 493–515

UPSCALING OF A CLASS OF NONLINEAR PARABOLIC EQUATIONS FOR THE FLOW TRANSPORT IN HETEROGENEOUS POROUS MEDIA*

ZHIMING CHEN†, WEIBING DENG‡, AND HUANG YE§

Abstract. We develop an upscaling method for the nonlinear parabolic equation

$$\partial_t b(u_\varepsilon) - \nabla\cdot(\mathbf{g}^\varepsilon(x,u_\varepsilon) + \mathbf{a}^\varepsilon(x,u_\varepsilon)\nabla u_\varepsilon) = f(x,t),$$

which stems from the applications of the flow transport in porous media. Our direct motivation is the Richards equation which models the flow transport in unsaturated porous media. We provide a detailed convergence analysis of the method under the assumption that the oscillating coefficients are periodic. While such a simplifying assumption is *not* required by our method, it allows us to use homogenization theory to obtain the asymptotic structure of the solutions. Numerical experiments are carried out for the Richards equation of exponential model with periodic and randomly generated log-normal permeability to demonstrate the efficiency and accuracy of the proposed method.

Key words. Upscaling, nonlinear parabolic equation, heterogeneous porous media.

AMS subject classifications. 65F10, 65F30

1. Introduction

Over the past three decades a significant amount of research effort has been devoted to determining what are known as effective parameters in the modelling of subsurface flow and transport. The central difficulty is the accounting for the spatial variability in the parameters used to characterize the relevant physical properties of the natural porous media. In realistic situations, it is impossible to account explicitly for the spatial variability at all scales, due to computational resource limitations and/or the lack of data. Thus sophisticated geological and geostatistical modelling tools are used in practice to generate highly detailed medium parameters based on some site-specific measurements and experience from other sites. There exists a vast literature on the upscaling or homogenization techniques that lump the small-scale details of the medium into a few representative macroscopic parameters or effective parameters on a coarse scale which preserve the large-scale behavior of the medium and are more appropriate for reservoir simulations. We refer to the book of Christakos [6] for more information on the random field modeling of the natural porous medium parameters and the recent review papers [27, 32] on the existent upscaling techniques in the engineering literature.

*Received: May 29, 2005; accepted (in revised version): August 11, 2005. Communicated by Shi Jin.
†LSEC, Institute of Computational Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100080, China (zmchen@lsec.cc.ac.cn).
This author was supported in part by China NSF under the grant 10025102 and by the National Basic Research Program of China under the grant 2005CB321701.
‡Department of Mathematics, Nanjing University, Nanjing 210093, China and LSEC, Institute of Computational Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100080, China (wbdeng@nju.edu.cn).
This author was supported in part by China Postdoctoral Science Foundation and the K. C. Wong Education Foundation, Hong Kong.
§LSEC, Institute of Computational Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, and Graduate School of the Chinese Academy of Sciences, Beijing 100080, China (yeh@lsec.cc.ac.cn).