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Spectral Decomposition of Path Space in Solvable Lattice Model

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Abstract: We give the *spectral decomposition* of the path space of the $U_q(\widehat{sl}_2)$ vertex model with respect to the local energy functions. The result suggests the hidden Yangian module structure on the $\widehat{sl}_2$ level l integrable modules, which is consistent with the earlier work [1] in the level one case. Also we prove the fermionic character formula of the $\widehat{sl}_2$ level l integrable representations in consequence.

1. Introduction

In the last decade of investigation, various close relations between the solvable lattice model and the conformal field theory have been revealed (for example, [2–5]). The aim of this article is to point out a new interesting relation between the spectrum in the solvable lattice model and the hidden quantum symmetry in the conformal field theory.

Consider the higher spin vertex model associated with the $l+1$ irreducible representation of $U_q(\widehat{sl}_2)$ ([6, 7]). It is well-known that the characters of the $\widehat{sl}_2$ or $U_q(\widehat{sl}_2)$ level l integrable representations $\mathscr{L}(k)$ can be calculated by using its *path space* $\mathscr{P}(k)$ ([2, 8]). The energy of a path $\vec{p}$ is given by the sum of a sequence of numbers $h(\vec{p}) = (h_1(\vec{p}), h_2(\vec{p}), \dots)$ minus the ground state energy which depends on the corresponding boundary condition. Here $h_i(\vec{p})$ is the i^{th} local energy determined from the $i+1^{\text{th}}$ component of $\vec{p}$ and its nearest neighbors by the local energy function. We propose the fact that the local energy functions not only play a combinatorial role, but also can be regarded as the $q \to 0$ limit of the local integrals of motion which commutes with the corner transfer matrix.

At $q = 0$, the energy of a path $\vec{p}$ is essentially the eigenvalue of the logarithm of the corner transfer matrix on the one dimensional configuration space $\sum_{\vec{p}\in\mathscr{P}(k)} \mathbf{C}\vec{p}$. Hence $\vec{p}$ itself is the "eigenvector" of the corner transfer matrix, and at the same time it is a simultaneous "eigenvector" of the mutually commuting infinitely many "local operators" h_i at $q = 0$.

In this paper we studied the *spectral decomposition* of the path space with respect to the local energy functions h_i. That is, we decomposed the path space