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Blow-up Results of Viriel Type for Zakharov Equations

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Abstract: We consider the Zakharov equations in $\mathbb{R}^N$ (for $N = 2, N = 3$). We first establish a viriel identity for such equations and then prove a blow-up result for solutions with a negative energy.

In this paper, we consider Zakharov equations in $\mathbb{R}^N$ (for $N = 2, 3$):

$$
\begin{aligned}
&iu_t = -\Delta u + nu \,, \\
(I'_{c_0}) \qquad &\frac{1}{c_0^2} n_{tt} = \Delta n + \Delta |u|^2 \,, \\
&u(0) = \phi_0,\ n(0) = n_0,\ n_t(0) = n_1 \,,
\end{aligned}
$$

where $c_0 > 0$, Δ is the Laplacian operator on $\mathbb{R}^N, u : [0,T) \times \mathbb{R}^N \to \mathbb{C}$, $n : [0,T) \times \mathbb{R}^N \to \mathbb{R}$ and ϕ_0, n_0, n_1 are initial data.

In fact, we consider equation (I'_{c_0}) in the Hamiltonian case. That is, we assume that there is a $w_0 : \mathbb{R}^N \to \mathbb{R}$ such that

$$n_t(0) = n_1 = -\Delta w_0 \,. \tag{1.1}$$

Then $\forall t$, there is a $w(t)$ such that

$$n_t(t) = -\Delta w(t) = -\nabla \cdot v(t) \,,$$

where $v(t) = \nabla w(t)$. In this case, (I'_{c_0}) can be written in the form

$$
\begin{aligned}
&iu_t = -\Delta u + nu \,, \\
&n_t = -\nabla \cdot v \,, \\
(I_{c_0}) \qquad &\frac{1}{c_0^2} v_t = -\nabla n - \nabla |u|^2 \,, \\
&u(0) = \phi_0, \qquad n(0) = n_0, \qquad v(0) = v_0 \,.
\end{aligned}
$$