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A Contribution of the Trivial Connection to the Jones Polynomial and Witten's Invariant of 3d Manifolds, II

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Abstract: We extend the results of our previous paper [1] from knots to links by using a formula for the Jones polynomial of a link derived recently by N. Reshetikhin. We establish a relation between the parameters of this formula and the multivariable Alexander polynomial. This relation is illustrated by an example of a torus link. We check that our expression for the Alexander polynomial satisfies some of its basic properties. Finally we derive a link surgery formula for the loop corrections to the trivial connection contribution to Witten's invariant of rational homology spheres.

1. Introduction

This paper is an expansion of our previous work [1]. We will try to extend the results of that paper from knots to links. Our main tool will be the formula for the Jones polynomial of a link proposed recently by N. Reshetikhin[2] [2].

We start by briefly reviewing the notations of [1] (they will be used throughout this paper) as well as some of its results. Let $\mathscr{L}$ be an n-component link in a 3-dimensional manifold M. We assign an α_j-dimensional $SU(2)$ representation to each component $\mathscr{L}_j$ of $\mathscr{L}$. E. Witten introduced in [3] an invariant $Z_{\alpha_1,\ldots,\alpha_n}(M,\mathscr{L};k)$ which is a path integral over the gauge equivalence classes of $SU(2)$ connection A_μ on M:

$$Z_{\alpha_1,\ldots,\alpha_n}(M,\mathscr{L};k) = \int[\mathscr{D}A_\mu]\exp\left(\frac{i}{\hbar}S_{CS}\right)\prod_{j=1}^{n}\mathrm{Tr}_{\alpha_j}\mathrm{Pexp}\left(\oint_{\mathscr{L}_j}A_\mu dx^\mu\right)\,, \tag{1.1}$$

here S_{CS} is the Chern–Simons action

$$S_{CS} = \frac{1}{2}\mathrm{Tr}\,\varepsilon^{\mu\nu\rho}\int_M dx(A_\mu\partial_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho)\,, \tag{1.2}$$

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