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Schwinger Terms and Cohomology of Pseudodifferential Operators

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Abstract: We study the cohomology of the Schwinger term arising in second quantization of the class of observables belonging to the restricted general linear algebra. We prove that, for all pseudodifferential operators in $3+1$ dimensions of this type, the Schwinger term is equivalent to the "twisted" Radul cocycle, a modified version of the Radul cocycle arising in non-commutative differential geometry. In the process we also show how the ordinary Radul cocycle for any pair of pseudodifferential operators in any dimension can be written as the phase space integral of the star commutator of their symbols projected to the appropriate asymptotic component.

1. Introduction

Current algebras play an important role in many quantum field theories. Historically, they were introduced in an attempt to describe hadronic processes. The hope was that the relevant physics would be captured by a restricted set of operators, the currents, satisfying linear commutation relations among themselves, and by a hamiltonian, bilinear in the currents, describing their time evolution. Even after the advent of QCD as the "microscopic" theory of strong interactions, physicists have often used current algebra techniques in the kinematical regions where the fundamental theory becomes intractable.

When seen from the point of view of a more fundamental theory, the currents are interpreted as composite operators in terms of the elementary fields, e.g. bilinears in some fermionic matter field. Often, at the quantum level, the naive conservation laws and commutation relations of the currents have to be modified by the addition of extra terms. In particular, when they spoil the conservation laws of some classically conserved current, these terms are referred to as anomalies. These anomalies are of crucial importance for the physical applications of the algebra; for global algebras

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