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Universal Drinfeld–Sokolov Reduction and Matrices of Complex Size

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Abstract: We construct affinization of the algebra $\mathfrak{gl}_\lambda$ of "complex size" matrices, that contains the algebras $\hat{\mathfrak{gl}}_n$ for integral values of the parameter. The Drinfeld–Sokolov Hamiltonian reduction of the algebra $\hat{\mathfrak{gl}}_\lambda$ results in the quadratic Gelfand–Dickey structure on the Poisson–Lie group of all pseudodifferential operators of complex order.

This construction is extended to the simultaneous deformation of orthogonal and symplectic algebras which produces self-adjoint operators, and it has a counterpart for the Toda lattices with fractional number of particles.

1. Introduction

As a rule quadratic Poisson structures appear as either the Poisson bracket on a Poisson–Lie group or as a result of Hamiltonian reduction from the linear bracket on a dual Lie algebra.

This paper is devoted to a relation between these two approaches to the classical W_n-algebras (called also Adler–Gelfand–Dickey or higher KdV-structures), natural infinite-dimensional quadratic Poisson structures on differential operators of n^{th} order.

The noncommutative Hamiltonian reduction (see [2, 22]) for the Gelfand–Dickey structures (associated to any reductive Lie group) is known as the reduction of Drinfeld and Sokolov ([7]). They showed that those quadratic structures on *scalar n^{th} order* differential operators on the circle can be obtained as a result of the two-step process (restriction to a submanifold and taking the quotient) from the linear Poisson structure on *matrix first order* differential operators. The latter object is nothing but the dual space to an affine Lie algebra on the circle ([13, 23]).

On the other hand all Poisson W_n algebras can be regarded as Poisson submanifolds in a certain universal Poisson–Lie group of pseudodifferential operators of arbitrary (complex) degree ([16]). In such a way differential operators $DO_n = \{D^n + u_1(x)D^{n-1} + u_2(x)D^{n-2} + \cdots + u_n(x)\}$ for any n turn out to be

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