On the Classification of Modular Fusion Algebras

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Abstract: We introduce the notion of (nondegenerate) strongly-modular fusion algebras. Here strongly-modular means that the fusion algebra is induced via Verlinde's formula by a representation of the modular group $\Gamma = \mathrm{SL}(2,\mathbb{Z})$ whose kernel contains a congruence subgroup. Furthermore, nondegenerate means that the conformal dimensions of possibly underlying rational conformal field theories do not differ by integers. Our main result is the classification of all strongly-modular fusion algebras of dimension two, three and four and the classification of all nondegenerate strongly-modular fusion algebras of dimension less than 24. We use the classification of the irreducible representations of the finite groups $\mathrm{SL}(2,\mathbb{Z}_{p^\lambda})$, where p is a prime and λ a positive integer. Finally, we give polynomial realizations and fusion graphs for all simple nondegenerate strongly-modular fusion algebras of dimension less than 24.

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