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## On $E_{10}$ and the DDF Construction

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Abstract: An attempt is made to understand the root spaces of Kac Moody algebras of hyperbolic type, and in particular  $E_{10}$ , in terms of a DDF construction appropriate to a subcritical compactified bosonic string. While the level-one root spaces can be completely characterized in terms of transversal DDF states (the level-zero elements just span the affine subalgebra), longitudinal DDF states are shown to appear beyond level one. In contrast to previous treatments of such algebras, we find it necessary to make use of a rational extension of the self-dual root lattice as an auxiliary device, and to admit non-summable operators (in the sense of the vertex algebra formalism). We demonstrate the utility of the method by completely analyzing a non-trivial level-two root space, obtaining an explicit and comparatively simple representation for it. We also emphasize the occurrence of several Virasoro algebras, whose interrelation is expected to be crucial for a better understanding of the complete structure of the Kac Moody algebra.

Subject Classifications: 17B67, 17B81, 81R10, 83E30

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