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The Fractal Volume of the Two-Dimensional Invasion Percolation Cluster

Yu Zhang★

Department of Mathematics, University of Colorado, Colorado Springs, CO 80933, USA

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Abstract. We consider both invasion percolation and standard Bernoulli bond percolation on the Z^2 lattice. Denote by $\mathscr{V}$ and $\mathscr{C}$ the invasion cluster and the occupied cluster of the origin, respectively. Let $\mathscr{V}_n = \mathscr{V} \cap [-n,n]^2$, and

$$\pi_n = P_{p_c}(\mathscr{C} \cap \text{ the boundary of } [-n,n]^2 \neq \emptyset)\,.$$

Let $\varepsilon > 0$ be given. Here we show that, with a probability tending to 1,

$$n^{2-\varepsilon}\pi_n \leqq |\mathscr{V}_n| \leqq n^{2+\varepsilon}\pi_n\,.$$

Assuming the existence of an exponent $1/\rho$ for π_n, it can be seen that with probability tending to one

$$n^{2-1/\rho-\varepsilon} \leqq |\mathscr{V}_n| \leqq n^{2-1/\rho+\varepsilon}\,.$$

Moreover, by den Nijs' and Nienhuis et al's computations,

$$n^{1.8958389583...-\varepsilon} = n^{1+\frac{43}{48}-\varepsilon} \leqq |\mathscr{V}_n| \leqq n^{1+\frac{43}{48}+\varepsilon} = n^{1.8958389583...+\varepsilon}$$

with a probability tending to one. The result matches Wilkinson and Willemsen's numerical computation $\mathscr{V}_n \sim n^{1.89}$. The method allows us also to show the same argument for any planar graph. Therefore, any two planar invasion clusters have the same fractal dimension $2 - \frac{1}{\rho}$ if one believes "universality."

Furthermore, the escape time of the invasion cluster is considered in this paper. More precisely, denote by h_n the first time that the invasion cluster escapes from $[-n,n]^2$. We here can show that with a probability tending to one

$$n^{2-\varepsilon}\pi_n \leqq h_n \leqq n^{2+\varepsilon}\pi_n\,.$$

Finally, invasion percolation with trapping is considered in this paper. Denote by

$$\mathscr{R}_n = \{\text{the number of bonds trapped by the invasion cluster before time } n\}\,.$$

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