Commun. Math. Phys. 165, 297–310 (1994)

Communications in Mathematical Physics

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Scalar Green's Functions in an Euclidean Space with a Conical-Type Line Singularity

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Received: 11 October 1993/in revised form: 10 February 1994

Abstract: In an Euclidean space with a conical-type line singularity, we determine the Green's function for a charged massive scalar field interacting with a magnetic flux running through the line singularity. We give an integral expression of the Green's function and a local form in the neighbourhood of the point source, where it is the sum of the usual Green's function in Euclidean space and a regular term. As an application, we derive the vacuum energy-momentum tensor in the massless case for an arbitrary magnetic flux.

1. Introduction

We consider an Euclidean space with a conical-type line singularity which is described by the metric

$$ds^2 = (dx^1)^2 + \cdots + (dx^{n-2})^2 + d\rho^2 + B^2\rho^2 d\varphi^2 \tag{1.1}$$

in a coordinate system (x^i, ρ, φ), $i = 1, \ldots, n-2$, such that $\rho \geq 0$ and $0 \leq \varphi < 2\pi$, the hypersurface $\varphi = 0$ and $\varphi = 2\pi$ being identified. Metric (1.1) is characterized by an arbitrary constant B, different from zero, and it is globally Euclidean for $B = 1$.

Riemannian metric (1.1) may result from the complexification of the time coordinate of a spacetime by a Wick rotation. In the Einstein theory of gravitation, this spacetime having a conical-type line source represents in three dimensions a point mass [1] and in four dimensions a straight cosmic string [2]; the constant B being determined by

$$B = 1 - \frac{4G}{c^2}\mu\,, \tag{1.2}$$

where μ is either the mass of the point mass or the linear mass density of the cosmic string.

⋆ Supported by a grant from CNPq (Brazilian government agency FA)