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The Stochastic Burgers Equation

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Abstract: We study Burgers Equation perturbed by a white noise in space and time. We prove the existence of solutions by showing that the Cole-Hopf transformation is meaningful also in the stochastic case. The problem is thus reduced to the anaylsis of a linear equation with multiplicative *half white* noise. An explicit solution of the latter is constructed through a generalized Feynman-Kac formula. Typical properties of the trajectories are then discussed. A technical result, concerning the regularizing effect of the convolution with the heat kernel, is proved for stochastic integrals.

1. Introduction

One of the first attempts to arrive at the statistical theory of turbulent fluid motion was the proposal by Burgers of his celebrated equation

$$\partial_t u_t(x) = \nu \partial_x^2 u_t(x) - u_t(x) \partial_x u_t(x) \,, \tag{1.1}$$

where $u_t(x)$ is the velocity field and ν is the viscosity. As Burgers emphasized in the introduction of his book [3] this equation represents an extremely simplified model describing the interaction of dissipative and non-linear inertial terms in the motion of the fluid. A clear discussion on the physical problems connected with Burgers equation can be found in [10]. As shown by Cole and Hopf [5, 7], Eq. (1.1) can be explicitly solved and, in the limit of vanishing viscosity, the solution develops shock waves.

Rigorous results have been recently established in the study of some statistical properties: random initial data are considered in [1, 14, 16], while in [15] a forcing term, which is a stationary stochastic process in time and a periodic function in space, is added.

The study of Burgers equation with a forcing term is interesting in view of the phenomenological character of (1.1). Since it represents an incomplete description of a system, a forcing term can provide a good model of the neglected effects; in particular a random perturbation may help to select interesting invariant measures.