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The Berezin Transform and Invariant Differential Operators

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Abstract: The Berezin calculus is important to quantum mechanics (creation-annihilation operators) and operator theory (Toeplitz operators). We study the basic Berezin transform (linking the contravariant and covariant symbol) for all bounded symmetric domains, and express it in terms of invariant differential operators.

0. Introduction

There are two equivalent ways to define the Wick calculus of operators on $\mathbb{R}^n$: the first one is based on creation and annihilation operators, a generalization of which constitutes a basic tool in quantum field theory. It is the alternative definition, based on reproducing kernel function theory, that leads to the generalization first defined and studied by Berezin [B1].

The Berezin calculus is of interest to theoretical physicists in that it constitutes a canonical quantization procedure associated with a fairly general class of phase spaces. Also, recent physics literature has shown much interest for coherent states, and Perelomov's book [P1] devotes a chapter to Berezin's theory. The operators obtained (also known as Toeplitz operators) generate some of the most interesting geometrically-structured C^*-algebras [BBCZ, U1]. Beyond applications to operator theory and partial differential equations [U8], Berezin operators are used in C^*-algebraic index theory and in deformation quantization of symmetric spaces [CGR, BLU].

Let M be a measure space, and let H be a closed subspace of $L^2(M)$, with orthogonal projection $E: L^2(M) \to H$. Then, given any bounded function f on M, the Berezin operator with contravariant symbol f is the linear operator $\sigma^*(f)$ on H given by

$$\sigma^*(f)h := E(fh). \tag{0.1}$$

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