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Bounds on the Growth of the Support of a Vortex Patch

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Abstract: We study the time evolution of the support of a vortex patch evolving in $\mathbb{R}^2$ according to the Euler Equation for an incompressible fluid and we bound its growth. Furthermore we discuss the same problem in the framework of a simplified model. Finally we consider a similar problem for the Navier-Stokes flow.

1. Introduction

In this paper we study the behavior of a non-viscous incompressible fluid in $\mathbb{R}^2$. In particular we consider the so-called vortex patch, that is a system in which the vorticity $\omega(x, 0)$ is proportional to the characteristic function χ of a region Λ_0:

$$\omega(x,0) = a\chi(\Lambda_0) \qquad a \in \mathbb{R}\,. \tag{1.1}$$

We suppose that initially Λ_0 has a bounded diameter $2R_0$. Then we evolve the vorticity by means of the Euler Equations: $\omega(x, 0) \to \omega(x, t)$. As it is well known $\omega(x, t)$ has the form:

$$\omega(x,t) = a\chi(\Lambda_t)\,. \tag{1.2}$$

Denote by $2R_t$ the diameter of Λ_t. We want to control its growth in time. This problem is interesting both for theoretical reasons, to understand deeply the dynamical behavior of the Euler equation and for applied ones (see for instance pollution problems).

In the present paper we want to find α, b for which we have:

$$R_t \leq (R_0^{1/\alpha} + bt)^\alpha \quad \text{for } t \geq 0\,. \tag{1.3}$$

It is trivial to observe that the boundedness of the velocity of the flow particles assures that Eq. (1.3) holds with $\alpha = 1$; however this bound is very bad and not so interesting. We want to improve this estimate. We observe that the main part of the

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