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Large Deviations for $\mathbb{Z}^d$-Actions

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Abstract: We establish large deviations bounds for translation invariant Gibbs measures of multidimensional subshifts of finite type. This generalizes [FO] and partially [C, O, and B], where only full shifts were considered. Our framework includes, in particular, the hard-core lattice gas models which are outside of the scope of [FO, C, O, and B].

1. Introduction

In [R1] Ruelle rewrote a part of the general theory of statistical mechanics for the case of a $\mathbb{Z}^d$-action, $d \geqq 1$ on a compact metric space Ω satisfying expansiveness and the specification. The main model for which one constructs translation invariant Gibbs states (see [R2]) consists of a finite set Q taken with the discrete topology and called the alphabet (which may represent, for instance, the spin values etc.), the set $Q^{\mathbb{Z}^d}$ considered with the product topology (making it compact) of all maps (configurations) $\omega : \mathbb{Z}^d \to Q$, the shifts $\theta_m, m \in \mathbb{Z}^d$ of $Q^{\mathbb{Z}^d}$ acting by the formula $(\theta_m \omega)_n = \omega_{n+m}$, where $\omega_k \in Q$ is the value of $\omega \in Q^{\mathbb{Z}^d}$ on $k \in \mathbb{Z}^d$, and a closed in the product topology subset Ω of $Q^{\mathbb{Z}^d}$ called the space of (permissible) configurations which is supposed to be shift invariant, i.e. $\theta_m \Omega = \Omega$ for every $m \in \mathbb{Z}^d$. The pair (Ω, θ) is called a subshift and if $\Omega = Q^{\mathbb{Z}^d}$ it is called the full shift. The construction in [R2] assumes, in fact, that (Ω, θ) is a subshift of finite type (see [Sh]) which means that there exist a finite set $F \subset \mathbb{Z}^d$ and a set $\Xi \subset Q^F$ such that

$$\Omega = \Omega_{(F,\Xi)} = \{\omega \in Q^{\mathbb{Z}^d} : (\theta_m \omega)_F \in \Xi \text{ for every } m \in \mathbb{Z}^d\}\,, \tag{1.1}$$

where $(\omega)_R = \omega_R$ denotes the restriction of $\omega \in Q^{\mathbb{Z}^d}$ to $R \subset \mathbb{Z}^d$. The set $\Xi \subset Q^F$ is the collection of permissible (allowed) words or configurations on F.

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