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Geometric Aspects of Quantum Spin States

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Abstract: A number of interesting features of the ground states of quantum spin chains are analyzed with the help of a functional integral representation of the system's equilibrium states. Methods of general applicability are introduced in the context of the SU($2S+1$)-invariant quantum spin-S chains with the interaction $-P^{(0)}$, where $P^{(0)}$ is the projection onto the singlet state of a pair of nearest neighbor spins. The phenomena discussed here include: the absence of Néel order, the possibility of dimerization, conditions for the existence of a spectral gap, and a dichotomy analogous to one found by Affleck and Lieb, stating that the systems exhibit either slow decay of correlations or translation symmetry breaking. Our representation elucidates the relation, evidence for which was found earlier, of the $-P^{(0)}$ spin-S systems with the Potts and the Fortuin–Kasteleyn random-cluster models in one more dimension. The method reveals the geometric aspects of the listed phenomena, and gives a precise sense to a picture of the ground state in which the spins are grouped into random clusters of zero total spin. E.g., within such structure the dichotomy is implied by a topological argument, and the alternatives correspond to whether, or not, the clusters are of finite mean length.

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