Two-Dimensional Topological Gravity and Equivariant Cohomology

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Abstract: The analogy between topological string theory and equivariant cohomology for differentiable actions of the circle group on manifolds has been widely remarked on. One of our aims in this paper is to make this analogy precise. We show that topological string theory is the "derived functor" of semi-relative cohomology, just as equivariant cohomology is the derived functor of basic cohomology. That homological algebra finds a place in the study of topological string theory should not surprise the reader, granted that topological string theory is the conformal field theorist's algebraic topology.

In [7], we have shown that the cohomology of a topological conformal field theory carries the structure of a Batalin–Vilkovisky algebra (actually, two commuting such structures, corresponding to the two chiral sectors of the theory). In the second part of this paper, we describe the analogous algebraic structure on the equivariant cohomology of a topological conformal field theory: we call this structure a gravity algebra. This algebraic structure is a certain generalization of a Lie algebra, and is distinguished by the fact that it has an infinite sequence of independent operations $\{a_1, \ldots, a_k\}$, $k \geq 2$, satisfying quadratic relations generalizing the Jacobi rule. (The operad underlying the category of gravity algebras has been studied independently by Ginzburg–Kapranov [9].)

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