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Roots of Unity: Representations of Quantum Groups

W.A. Schnizer★†

RIMS, Kyoto University, Kyoto 606, Japan and Institute for Nuclear Physics, TU-Wien, 1040 Wien, Austria

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Abstract: Representations of Quantum Groups $U_\varepsilon(g_n)$, g_n any semi-simple Lie algebra of rank n, are constructed from arbitrary representations of rank $n-1$ quantum groups for ε a root of unity. Representations which have the maximal dimension and number of free parameters for irreducible representations arise as special cases.

1. Introduction

Deformations of semi-simple Lie algebras [18, 19] appear as a common algebraic structure in the field of low dimensional integrable systems. In many cases the deformation parameter is an N-th root of unity, where N can correspond, e.g. to the number of states per site or to the lattice size in a two dimensional model. We will denote the deformation parameter by ε, if the parameter is an N-th root of unity (N the smallest integer such that $\varepsilon^N = 1$) and by q in the general case.

The theories of chiral Potts [4, 5] type models, which saw dramatic developments in recent years [6–8, 12, 24], are closely tied to the representation theory of the quantum group $U_\varepsilon(sl(n, \mathbf{C}))$ in the case of ε being an N-th root of unity. The progress in the theories of chiral Potts models was partly stimulated by the better understanding of its deep connection to the representation theory of quantum groups.

The representation theory in the case of ε an N-th root of unity is much richer than for generic q, and several deep results by De Concini, Kac, Procesi [15, 17, 16] and Lusztig [20–23] exist, laying the foundations of the general representation theory in the roots of unity case. Also considerable progress has been made in directly constructing representations of quantum groups $U_\varepsilon(g_n)$. Accelerated by the development of chiral Potts type models, much interest was devoted to find non-highest weight representations of $U_\varepsilon(g_n)$ [25, 1–3, 9–11]. Finite dimensional

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† Deceased