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Fermion Current Algebras and Schwinger Terms in (3 + 1)-Dimensions

Edwin Langmann*

Department of Physics, The University of British Columbia, Vancouver, B.C., Canada V6T 1Z1

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Abstract: We discuss the restricted linear group in infinite dimensions modeled by the Schatten class of rank $2p = 4$ which contains the $(3 + 1)$-dimensional analogs of the loop groups and is closely related to Yang–Mills theory with fermions in $(3 + 1)$-dimensions. We give an alternative to the construction of the "highest weight" representation of this group found by Mickelsson and Rajeev. Our approach is close to quantum field theory, with the elements of this group regarded as Bogoliubov transformations for fermions in an external Yang–Mills field. Though these cannot be unitarily implemented in the physically relevant representation of the fermion field algebra, we argue that they can be implemented by sesquilinear forms, and that there is a (regularized) product of forms providing an appropriate group structure. On the Lie algebra level, this gives an explicit, non-perturbative construction of fermion current algebras in $(3 + 1)$ space-time dimensions which explicitly shows that the "wave function renormalization" required for a consistent definition of the currents and their Lie bracket naturally leads to the Schwinger term identical with the Mickelsson–Rajeev cocycle. Though the explicit form of the Schwinger term is given only for the case $p = 2$, our arguments apply also to the restricted linear groups modeled by Schatten classes of rank $2p = 6, 8, \ldots$ corresponding to current algebras in $(d + 1)$-dimensions, $d = 5, 7, \ldots$.

1. Introduction

In recent years it has become obvious that the representation theory of certain infinite dimensional Lie groups and algebras can contribute in an essential way to the understanding of quantum field theory. Well-known examples are the Virasoro algebra and the affine Kac–Moody algebras which have been of crucial importance for recent spectacular progress in two dimensional conformal field theory.

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