Commun. Math. Phys. 159, 1–13 (1994)

Communications in
**Mathematical
Physics**

Irreducible Representations of Virasoro-Toroidal Lie Algebras

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Received: 16 November 1992/in revised form: 10 May 1993

To A. John Coleman on the occasion of his 75th birthday

Abstract: Toroidal Lie algebras and their vertex operator representations were introduced in [MEY] and a class of indecomposable modules were investigated. In this work, we extend the toroidal algebra by the Virasoro algebra thus constructing a semi-direct product algebra containing the toroidal algebra as an ideal and the Virasoro algebra as a subalgebra. With the use of vertex operators and certain oscillator representations of the Virasoro algebra it is proved that the corresponding Fock space gives rise to a class of irreducible modules for the Virasoro-toroidal algebra.

Introduction

Toroidal algebras $\mathfrak{t}_{[n]}$ are defined for every $n \geqq 1$ and when $n = 1$ they are precisely the untwisted affine algebras. Such an affine algebra $\mathfrak{g}$ can be realized as the universal covering algebra of the loop algebra $\dot{\mathfrak{g}} \otimes_{\mathbb{C}} \mathbb{C}[t, t^{-1}]$ where $\dot{\mathfrak{g}}$ is a simple finite dimensional Lie algebra over $\mathbb{C}$. It is well known that $\mathfrak{g}$ is a one-dimensional central extension of $\dot{\mathfrak{g}} \otimes_{\mathbb{C}} \mathbb{C}[t, t^{-1}]$. The toroidal algebras $\mathfrak{t}_{[n]}$ are the universal covering algebras of iterated loop algebras $\dot{\mathfrak{g}} \otimes_{\mathbb{C}} \mathbb{C}[t_1^{\pm}, \ldots, t_n^{\pm}]$ which, for $n \geqq 2$, turn out to be infinite-dimensional central extensions.

Unlike the finite dimensional case, there is a distinguished irreducible highest weight module for any untwisted (or direct) affine Lie algebra. This is the *basic representation.* In 1980 Frenkel and Kac [FK] gave a remarkable construction of the basic representation by using vertex operators $X(\alpha, z)$, where α runs over the root lattice $\dot{Q}$ of $\dot{\mathfrak{g}}$. Already in [FK] it was observed that the Virasoro algebra also operates on the basic representation and in particular the (energy) operator d_0 plays a distinguished role.

A decade later, the vertex operators $X(\alpha, z)$, where α now lies in the affine root lattice, $Q = \dot{Q} \oplus \mathbb{Z}\delta$, were used to produce indecomposable representations of the toroidal algebras $\mathfrak{t}_{[2]}$ [MEY]. Soon after, these results were shown [EM] to extend to arbitrary n.

However these representations are not completely reducible, nor do irreducible representations appear in a natural way in the picture. The objective of this paper is