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A Sharp Lower Bound for the Hausdorff Dimension of the Global Attractors of the 2D Navier-Stokes Equations

Vincent Xiaosong Liu

Department of Mathematics, Stanford University, Stanford, CA 94305-2125, USA

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Abstract. For a special class of the Navier-Stokes equations on the two-dimensional torus, we give a lower bound in the form $G^{2/3}$ (where G is the Grashof number) for the Hausdorff dimension of its global attractor which is optimal up to a logarithmic term.

1. Preliminaries and Introduction

We continue our previous work [8, 9] on the 2D Navier-Stokes equations for a viscous incompressible fluid with spatially periodic boundary conditions. In [9], we get a lower bound for the Hausdorff dimension of the global attractor in the form $G^{1/3}$ by considering some unstable modes of the associated linear operator for the Navier-Stokes equations. We commented there that one can improve the lower bound by more careful examination. In this paper, following the same technique as in [8, 9], we improve the lower bound to $G^{2/3}$ by considering more unstable eigenmodes. The idea is simple, since the dimension of unstable manifold around a steady state gives a lower bound for the Hausdorff dimension of the global attractor, so we only need to give an estimate for the number of unstable directions around this steady state.

Navier-Stokes equations written in functional form are [4, 15, 16]:

$$\frac{du}{dt} + Au + B(u,u) = f, \tag{1}$$

$$u(0) = u_0\,, \tag{2}$$

Dedicated to the memory of Professor Nicholas D. Kazarinoff