Commun. Math. Phys. 157, 23–50 (1993)

Communications in Mathematical Physics


# Spectral Properties of Random Schrödinger Operators with Unbounded Potentials

**Y.A. Gordon**[1], **V. Jakšić**[2★], **S. Molčanov**[1], **and B. Simon**[3★★]

[1] Department of Mathematics and Mechanics, Moscow State University, Moscow 119808, Russia
[2] Department of Mathematics, University of Toronto, Toronto, M5S-1A1, Canada
[3] Division of Physics, Mathematics and Astronomy, California Institute of Technology, Pasadena, CA 91125, USA

Received May 25, 1992

**Abstract.** We investigate spectral properties of random Schrödinger operators $H_\omega = -\Delta + \xi_n(\omega)(1 + |n|^\alpha)$ acting on $l^2(Z^d)$, where $\xi_n$ are independent random variables uniformly distributed on [0, 1].

## 1. Introduction

It is already a part of folklore that multiplicative perturbations of the Anderson model show rather "unusual" spectral behavior. The basic paradigm is the discrete Schrödinger operator on $l^2(Z^1)$,

$$H_\omega u(n) = 2u(n) - u(n+1) - u(n-1) + V_\omega(n)u(n) ,$$

$$V_\omega(n) = \lambda \xi_n(\omega)|n|^\alpha ,$$

where $\xi_n(\omega)$ are independent random variables with a bounded, compactly supported density $r(x)$, and $\lambda$ is a parameter. For $\alpha < 0$ the above model has been extensively studied in [5, 7, 8, 18] and their main results can be summarized as follows (note that for $\alpha < 0$, $V_\omega(n) \to 0$ as $|n| \to \infty$ and thus $\sigma_{\mathrm{ess}}(H_\omega) = [0, 4]$).

**Theorem.** *With probability* 1:

(i) *For* $-1/2 < \alpha < 0$, *the spectrum in* $[0, 4]$ *is pure point with eigenfunctions decaying as* $\exp(-C|n|^{1+2\alpha})$.
(ii) *For* $\alpha < -1/2$, *the spectrum in* $[0, 4]$ *is purely absolutely continuous.*
(iii) *For* $\alpha = -1/2$ *and* $\lambda$ *large, the spectrum in* $[0, 4]$ *is pure point with polynomially decaying eigenfunctions, while for* $\lambda$ *small* $H_\omega$ *will have some singular continuous spectrum.*

---

★ Research partially supported by a Sloan Doctoral Dissertation Fellowship and NSERC under grant OGP-0007901
★★ Research partially supported by NSF grant DMS-9101716