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Geometry of Geometrically Finite One-Dimensional Maps

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Abstract. We study the geometry of certain one-dimensional maps as dynamical systems. We prove the property of bounded and bounded nearby geometry of certain $C^{1+\alpha}$ one-dimensional maps with finitely many critical points. This property enables us to give the quasisymmetric classification of geometrically finite one-dimensional maps.

Contents

1. Introduction

Two smooth maps f and g from a one-dimensional C^2-Riemannian manifold M into itself are topologically conjugate if there is a homeomorphism h from M onto itself such that $f \circ h = h \circ g$. A nontrivial problem [10] asked by Sullivan was about whether the conjugating map h is necessarily quasisymmetric [1]. (We note that when f and g are both holomorphic and expanding maps on a domain in the Riemann sphere, then h is quasiconformal [1] because of bounded geometry property [9, 10].) In [4 and 5], we studied this kind of problem for an interval map with one critical point. In this paper, we generalize the results of [4 and 5] to geometrically finite maps which are certain one-dimensional maps with finitely many critical points (see Definition 3 in Sect. 2). We prove that the induced sequence of nested partitions of M by a geometrically finite map (see Sect. 2 for the definition) has bounded and bounded nearby geometry (see Definition 2 in Sect. 2).