The Operator Quantization
of the Open Bosonic String:
Field Algebra

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Abstract. Our aim in this paper is to make explicit the operator theory of the heuristic open Bosonic string and to abstract a suitable field algebra for the string. This is done on a Fock–Krein space and we examine integrability and J-unitary implementability of all the defining transformations of the string, i.e. time translations, gauge transformations and Poincaré transformations. The results obtained agree partially with those of Bowick and Rajeev, i.e. the gauge transformations do not leave the Fock–Krein complex structure invariant. Once we obtained integrated transformation groups on a suitable symplectic space for the infinitesimal transformations of the string, and proved implementability of these for the Fock–Krein representation, we are then free to define an abstract C*-algebra carrying all the algebraic information of the string, and to examine different representations.

Introduction

There are several rigorous approaches to the open bosonic quantum string, of which we find the geometric approach of Bowick and Rajeev [16, 17] and Mickelsson [15] most appealing. This approach produced the following results:

 (i) The complex structure K which defines the quantum Hilbert space for the string, is not invariant under the gauge group $\text{Diff}^+ S^1$.
 (ii) The orbit of K under $\text{Diff}^+ S^1$ is taken as the dynamical manifold, endowed with the topology of the homogeneous space $\text{Diff}^+ S^1/S^1$ which is bijective to it. This manifold has a Kähler (hence symplectic) structure, on which one constructs a Fock bundle $\mathscr{B}$ using the complex structure at each point to obtain the one particle spaces. The curvature of this bundle produces the Virasoro anomaly as a two-cocycle. Mickelsson [15] showed that $\mathscr{B}$ has no $\text{Diff}^+ S^1$ invariant sections, but it is possible to adjoin a fermionic ghost bundle to $\mathscr{B}$ which produces a bundle with such sections in dimension 26.

However, geometric quantization is not yet a full quantization, so there is still some mathematical distance between the heuristic quantum string and the geometric