

Radiation Conditions and Scattering Theory for N -Particle Hamiltonians

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Received September 9, 1992; in revised form November 20, 1992

Abstract. The correct form of the angular part of radiation conditions is found in scattering problem for N -particle quantum systems. The estimates obtained allow us to give an elementary proof of asymptotic completeness for such systems in the framework of the theory of smooth perturbations.

1. Introduction

One of the main problems of scattering theory is a description of asymptotic behaviour of N interacting quantum particles for large times. The complete classification of all possible asymptotics (channels of scattering) is called asymptotic completeness. The final result can easily be formulated in physics terms. Two particles can either form a bound state or are asymptotically free. In the case $N \geq 3$ a system of N particles can additionally be decomposed for large times into non-trivial subsystems (clusters). Particles of the same cluster form a bound state and different clusters do not interact with each other.

There are two essentially different approaches to a proof of asymptotic completeness for multiparticle ($N \geq 3$) quantum systems. The first of them, suggested by L.D. Faddeev [1], relies on the detailed study of a set of equations derived by him for the resolvent of the corresponding Hamiltonian. This approach was developed in [1] for the case of three particles and was further elaborated by J. Ginibre and M. Moulin [2] and L. Thomas [3]. The attempts [4, 5] towards a straightforward generalization of Faddeev's method to an arbitrary number of particles meet with numerous difficulties. However, the results of [6] for weak interactions are quite elementary.

Another approach relies on the commutator method [7] of T. Kato. In the theory of N -particle scattering it was introduced by R. Lavine [8, 9] for repulsive potentials. The proof of asymptotic completeness in the general case is much more complicated and is due to I. Sigal and A. Soffer [10] (see also the article [11] by

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