Commun. Math. Phys. 153, 147–158 (1993)

Communications in Mathematical Physics

Nonlinear Stability of Overcompressive Shock Waves in a Rotationally Invariant System of Viscous Conservation Laws

Heinrich Freistühler[1] **and Tai-Ping Liu**[2]

[1] Institut für Mathematik, RWTH Aachen, W-5100 Aachen, Germany. Research supported by Deutsche Forschungsgemeinschaft

[2] Mathematics Department, Stanford University, CA 94305, USA. Research supported in part by NSF Grant DMS 90-0226 and Army Grant DAAL 03-91-G-0017

Received August 27, 1992

Abstract. This paper proves that certain non-classical shock waves in a rotationally invariant system of viscous conservation laws possess nonlinear large-time stability against sufficiently small perturbations. The result applies to small intermediate magnetohydrodynamic shocks in the presence of dissipation.

1. Introduction

In this paper, we consider the parabolic system

$$u_t + (|u|^2 u)_x = \mu u_{xx} \,, \tag{1.1}$$

where $x \in \mathbb{R}$, $t \in \mathbb{R}$, $u(x, t) \in \mathbb{R}^n$ $(n \geqq 2)$, $\mu > 0$, and study the question of large time stability of some of its shock wave solutions

$$u_*(x, t) = \phi_*((x - st)\mu), \quad \phi_*(\pm\infty) = u^\pm, \quad u^- \neq u^+ \,. \tag{1.2}$$

System (1.1) and these shock waves have physically relevant interpretations as we will detail soon below. The system is rotationally invariant and thus its inviscid part

$$u_t + (|u|^2 u)_x = 0 \tag{1.3}$$

is *non-strictly* hyperbolic: The characteristic speeds

$$\lambda_1(u) = |u|^2, \quad \lambda_2(u) = 3|u|^2 \tag{1.4}$$

touch at the umbilic point 0; the corresponding eigenspaces

$$R_i(u) = \ker((|u|^2 - \lambda_i(u))I + 2uu^t), \quad i = 1, 2 \,,$$

rotate as

$$R_1(u) = \{u\}^\perp, \quad R_2(u) = \mathbb{R}u, \quad u \neq 0 \,, \tag{1.5}$$