Commun. Math. Phys. 153, 77-115 (1993)



## On a Ferromagnetic Spin Chain

## Andreas Knauf

Technische Universität, Fachbereich 3 – Mathematik, MA 7–2, Strasse des 17. Juni 135, W-1000 Berlin 2, Germany

Received May 28, 1992; in revised form August 19, 1992

Abstract. The quotient  $\zeta(s-1)/\zeta(s)$  of Riemann zeta functions is shown to be the partition function of a ferromagnetic spin chain for inverse temperature s.

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## 1. Introduction

The aim of this article is to relate ideas and concepts from statistical mechanics to the Riemann zeta function.

The quotient  $Z(s) = \zeta(s-1)/\zeta(s)$  is interpreted as the partition function of an infinite ferromagnetic spin chain.

The existence of a connection between number theory and statistical mechanics has been conjectured by Kac (see his Comments in Pólya [4], pp. 424–426), Newman [3], Ruelle [5] and others.

One motivation for that conjecture has been the Lee-Yang circle theorem of statistical mechanics. In its basic form it states that all zeroes of the partition function of a ferromagnetic Ising model in the complex activity plane have unit modulus.