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Poisson Measure Representation and Cluster Expansion in Classical Statistical Mechanics

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Abstract. A new representation for distribution functions of the grand canonical ensemble by the Poisson measure functional integral is obtained. Due to the ultralocal nature of the measure, the construction of the cluster expansion is very simple. For the convergence of the cluster expansion, the requirement of exponential decay of the interaction potential is not necessary.

1. Introduction

The cluster expansion method, proposed by Glimm, Jaffe, and Spencer [1] for the investigation of quantum field theory models, was greatly developed by Brydges and Federbush [2, 3] in statistical mechanics. It was especially fruitful in the classical statistical mechanics for the study of screening effects in charged particle systems [4–9]. However, the range of applicability of this method is restricted to exponentially decreasing interactions, and in addition, the technique of construction of cluster expansions is very complicated. The requirement of the exponential decay is needed to compensate the large powers of $N!$, where N is connected with the order of decomposition. These factorials appear especially due to variational derivatives over Gaussian fields (sine-Gordon variables). In turn, the variational derivatives arise on each step of decomposition in the formula of replacing the initial Gaussian measure by the new one which provides the factorization of Gaussian integrals (see [1, 4, 5, 10] for details).

In this paper, we propose a new representation for distribution functions of the grand canonical ensemble. This representation gives an opportunity to simplify considerably the construction of the cluster expansion and the proof of its convergence, and also extends the set of admissible potentials up to the class of integrable functions (or for slightly stronger condition). This representation is based on the fact that the expression which is obtained after the sine-Gordon transformation and summing up

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