Commun. Math. Phys. 149, 361–376 (1992)

Communications in Mathematical Physics

The Existence of Non-Topological Solitons in the Self-Dual Chern–Simons Theory

Joel Spruck [1,*] **and Yisong Yang** [2]

[1] Department of Mathematics, University of Massachusetts, Amherst, MA 01003, USA and I.H.E.S, Bures-Sur-Yvette, France
[2] Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA

Received December 6, 1991; in revised form March 10, 1992

Abstract. In the recently discovered $(2+1)$-dimensional relativistic Chern–Simons model, self-duality can be achieved when the Higgs potential density assumes a special form for which both the asymmetric and symmetric vacua are ground state solutions. This important feature may imply the coexistence of static topological and non-topological vortex-like solutions in $\mathbb{R}^2$ but the latter have been rather elusive to a rigorous construction. Our main purpose in this paper is to prove the existence of non-topological radially symmetric N-vortex solutions in the self-dual Chern–Simons model. By a shooting method, we obtain a continuous family of gauge-distinct N-vortex solutions. Moreover, we are also able to classify all possible bare (or 0-vortex) solutions.

1. Introduction

Although it has long been speculated that the addition of a Chern–Simons term to the usual Yang–Mills–Higgs action density in $(2+1)$ dimensions would lead to static finite energy vortex-like solutions that carry both electric and magnetic charges, it is the work of Hong, Kim, and Pac [10] and Jackiw and Weinberg [15] (see also Lee [17] for the nonabelian case) which enables one to make a rigorous study of such solutions. The crucial feature in their approach is that, when the Yang–Mills (or Maxwell) term is dropped from the Lagrangian and the Higgs potential takes on a special form, the static equations of motion can be reduced to a new Bogomol'nyi type system of first order equations. Due to the form of the Higgs potential, both topological and non-topological solutions may be present [11–14]. In fact, the system shares some common properties with the Bogomol'nyi equations arising from the abelian Higgs model [5] and hence, partial understanding may be achieved using some techniques developed earlier. In particular, Wang [25] has proved the existence of topological multivortex solutions by the variational method in Jaffe and Taubes [16]. Moreover,

* Research supported in part by NSF grant DMS-88-02858 and DOE grant DE-FG02-86ER250125